

Isolating genuine nonclassicality in tripartite quantum correlations

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We introduce the measures, Svetlichny and Mermin discord, to characterize the presence of genuine nonclassicality in tripartite quantum correlations. We show that any correlation in the Svetlichny-box polytope which is a subpolytope of full tripartite nonsignaling polytope admits a three-way decomposition using these measures of nonclassicality. This decomposition allows us to isolate the origin of nonclassicality into three disjoint sources: a Svetlichny box which exhibits three-way nonlocality, a maximally two-way nonlocal box which exhibits three-way contextuality, and a classical correlation. Svetlichny and Mermin discord quantify three-way nonlocality and three-way contextuality of quantum correlations with respect to the three-way decomposition in that nonzero value of these measures imply the presence of noncommuting measurements that give rise to these two forms of genuine nonclassicality.

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I. INTRODUCTION

Correlations between outcomes of local measurements on entangled states are in general incompatible with local hidden variable (LHV) theories [1]. In the multipartite scenario, distinct types of LHV theories exist [2]. In the tripartite case, Svetlichny showed that quantum correlations can have genuine nonlocality which cannot be explained by hybrid local-nonlocal hidden variable (HLHV) theory [3]. Just like bipartite quantum correlations cannot violate a Bell-CHSH inequality more than the Tsirelson bound [2], multipartite quantum correlations cannot violate a Svetlichny inequality more than a certain bound [4]. Quantum theory is only a subclass of a multipartite generalized nonsignaling theory that predicts extremal genuine nonlocality [5]. Generalized nonsignaling theories have been under investigation to find out what physical principles exactly captures quantum correlations in addition to nonsignaling principle and nonlocality [2, 6]. In Ref. [7], it was shown that a complete characterization of quantum correlations requires genuine multipartite principles. Genuine multipartite nonlocality is a resource for multipartite quantum information tasks [8]. Thus, characterizing and quantifying multipartite correlations using genuine multipartite concepts is of interest to both foundations and quantum information.

Recently, it has been shown that there are mixed separable states that give rise to advantage for certain bipartite quantum information tasks [9, 10]. The key resource behind this advantage is known to be quantum discord which was introduced as a measure of quantum correlations [11, 12]. Georgi *et al.* [13] defined genuine quantum discord to quantify nonclassical correlation in tripartite quantum systems that cannot be reduced to the correlation present in any possible subsystems.

In this work, we characterize the presence of genuine tripartite nonclassicality in boxes, i.e., probability distributions arising from local measurements on tripartite quantum states by using two binary inputs and two binary outputs nonsignaling

(NS) polytope [14]. We define Svetlichny and Mermin discord using Svetlichny and Mermin operators which put an upper bound on the correlations under the constraints of the HLHV model [3] and fully LHV model [15]. Analogous to genuine quantum discord [13], these measures detect the presence of genuine quantum nonclassicality in Svetlichny-local boxes as well. We obtain a 3-decomposition of any box in the Svetlichny-box polytope which is a subpolytope of the full NS polytope by using Svetlichny and Mermin discord. This decomposition allows us to isolate the origin of genuine nonclassicality of quantum correlations into three disjoint sources: a Svetlichny box which exhibits three-way nonlocality, a maximally two-way nonlocal box which exhibits three-way contextuality and a classical correlation. We apply Svetlichny and Mermin discord to boxes arising from various three-qubit states. We notice that a nonzero value of Svetlichny and Mermin discord of these boxes originate from noncommuting measurements that give rise to three-way nonlocality and three-way contextuality, respectively. We identify the set of genuinely tripartite nonclassical biseparable and fully separable three-qubit states using Svetlichny and Mermin discord.

II. PRELIMINARIES

Consider the Bell scenario in which three spatially separated parties, Alice, Bob and Charlie, share a tripartite box which has two binary inputs and two binary outputs per party. The correlation between the outputs is captured by the set of joint probability distributions (JPDs), $P(a_m, b_n, c_o | A_i, B_j, C_k)$, here $m, n, o, i, j, k \in \{0, 1\}$. In addition to positivity and normalization, the JPDs characterizing a given box satisfy nonsignaling constraints:

$$\sum_m P(a_m, b_n, c_o | A_i, B_j, C_k) = P(b_n, c_o | B_j, C_k) \quad \forall n, o, i, j, k, \quad (1)$$

and the permutations. The set of such NS boxes forms a convex polytope, \mathcal{N} , in a 26 dimensional space [16]. Any box that belongs to this polytope can be uniquely described by 6

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single-party, 12 two-party and 8 three-party expectations as follows,

$$P(a_m, b_n, c_o | A_i, B_j, C_k) = \frac{1}{8} [1 + (-1)^m \langle A_i \rangle + (-1)^n \langle B_j \rangle + (-1)^o \langle C_k \rangle + (-1)^{m \oplus n} \langle A_i B_j \rangle + (-1)^{m \oplus o} \langle A_i C_k \rangle + (-1)^{n \oplus o} \langle B_j C_k \rangle + (-1)^{m \oplus n \oplus o} \langle A_i B_j C_k \rangle]. \quad (2)$$

Pironio *et al.* [14] found that \mathcal{N} has 53856 extremal boxes (vertices) which belong to 46 classes. The vertices in each class are equivalent in that they can be converted into each other through local reversible operations (LRO), which include local relabeling of the inputs and outputs [16]. These 46 classes of vertices can be classified into local, two-way nonlocal and 44 classes of three-way nonlocal vertices.

Two-way local polytope, \mathcal{L}_2 , is a convex subpolytope of \mathcal{N} whose vertices are the 64 local vertices and the 48 two-way nonlocal vertices. The local vertices are fully deterministic boxes given as follows,

$$P_D^{\alpha\beta\gamma\epsilon\zeta\eta}(a_m, b_n, c_o | A_i, B_j, C_k) = \begin{cases} 1, & m = \alpha i \oplus \beta \\ & n = \gamma j \oplus \epsilon \\ & o = \zeta k \oplus \eta \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

Here $\alpha, \beta, \gamma, \epsilon, \zeta, \eta \in \{0, 1\}$ and \oplus denotes addition modulo 2. The two-way nonlocal vertices are the bipartite PR-boxes: there are 16 vertices in which PR-box is shared between A and B ,

$$P_{12}^{\alpha\beta\gamma\epsilon}(a_m, b_n, c_o | A_i, B_j, C_k) = \begin{cases} \frac{1}{2}, & m \oplus n = i \cdot j \oplus \alpha i \oplus \beta j \oplus \gamma & \& \quad o = \epsilon k \\ 0, & \text{otherwise,} \end{cases} \quad (4)$$

and the other 32 two-way nonlocal vertices, $P_{13}^{\alpha\beta\gamma\epsilon}$ and $P_{23}^{\alpha\beta\gamma\epsilon}$, in which PR-box is shared by AC and BC are similarly defined. \mathcal{L}_2 can be divided into a two-way nonlocal region and Bell-local polytope, \mathcal{L} , whose vertices are the deterministic boxes given in Eq. (3). All correlations in \mathcal{L} can be explained by the LHV theory, i.e., the correlations can be decomposed as follows,

$$P(a_m, b_n, c_o | A_i, B_j, C_k) = \sum_{\lambda} p_{\lambda} P_{\lambda}(a_m | A_i) P_{\lambda}(b_n | B_j) P_{\lambda}(c_k | C_k), \quad (5)$$

whereas all correlations in the two-way nonlocal region can be decomposed into the hybrid local-nonlocal form in which arbitrary nonlocality consistent with nonsignaling principle is allowed between two parties in the different bipartitions,

$$P(a_m, b_n, c_o | A_i, B_j, C_k) = p_1 \sum_{\lambda} p_{\lambda} P_{\lambda}^{A|B|C} + p_2 \sum_{\lambda} q_{\lambda} P_{\lambda}^{A|C|B} + p_3 \sum_{\lambda} r_{\lambda} P_{\lambda}^{A|BC}, \quad (6)$$

where $P_{\lambda}^{A|B|C} = P_{\lambda}(a_m, b_n | A_i, B_j) P_{\lambda}(c_o | C_k)$, and, where $P_{\lambda}^{A|C|B}$ and $P_{\lambda}^{A|BC}$ are similarly defined.

Nonlocal boxes that do not admit the decomposition in Eq. (6) exhibit genuine three-way nonlocality. Three-way nonlocal boxes violate a facet inequality corresponding to \mathcal{L}_2 . Bancal *et al.* [17] found that \mathcal{L}_2 has 185 classes of facet inequalities. In this work, we consider two classes of 3-way nonlocal vertices that belong to the classes 8 and 46 given in Pironio *et al.* [14]. The extremal boxes that belong to the class 8 violate a class 99 facet inequality to its algebraic maximum. A representative of the class 99 facet inequality is given by,

$$\mathcal{L}_2^{99} = \langle A_0 B_0 \rangle + \langle A_0 C_0 \rangle + \langle B_1 C_0 \rangle + \langle A_1 B_0 C_1 \rangle - \langle A_1 B_1 C_1 \rangle \leq 3. \quad (7)$$

The representative of class 8 extremal box given in the table of Ref. [14] has $\langle A_0 B_0 \rangle = \langle A_0 B_1 \rangle = \langle A_0 C_0 \rangle = \langle B_0 C_0 \rangle = \langle B_1 C_0 \rangle = \langle A_1 B_0 C_1 \rangle = -\langle A_1 B_1 C_1 \rangle = 1$ and the rest of the expectations are zero which imply $\mathcal{L}_2^{99} = 5$. The extremal boxes that belong to the class 46 are 16 Svetlichny-boxes,

$$P_{\text{Sv}}^{\alpha\beta\gamma\epsilon}(a_m, b_n, c_o | A_i, B_j, C_k) = \begin{cases} \frac{1}{4}, & m \oplus n \oplus o = i \cdot j \oplus i \cdot k \oplus j \cdot k \oplus \alpha i \oplus \beta j \oplus \gamma k \oplus \epsilon \\ 0, & \text{otherwise,} \end{cases} \quad (8)$$

which violate one of the class 185 facet inequalities,

$$S_{\alpha\beta\gamma\epsilon} = \sum_{ijk} (-1)^{i \cdot j \oplus i \cdot k \oplus j \cdot k \oplus \alpha i \oplus \beta j \oplus \gamma k \oplus \epsilon} \langle A_i B_j C_k \rangle \leq 4, \quad (9)$$

to its algebraic maximum of 8. A class 185 facet inequality is known as Svetlichny inequality [3]. We will refer to the boxes which do not violate a Svetlichny inequality as Svetlichny-local.

In this work, we consider quantum correlations arising from Svetlichny scenario [3] in which the parties generate the JPDs by making spin projective measurements $A_i = \hat{a}_i \cdot \vec{\sigma}$, $B_j = \hat{b}_j \cdot \vec{\sigma}$ and $C_k = \hat{c}_k \cdot \vec{\sigma}$ on an ensemble of three-qubit system described by the density matrix ρ in the Hilbert space $\mathcal{H}_2^A \otimes \mathcal{H}_2^B \otimes \mathcal{H}_2^C$. The correlation predicted by quantum theory is defined as follows,

$$P(a_m, b_n, c_o | A_i, B_j, C_k) = \text{Tr}(\rho \Pi_{A_i}^{a_m} \otimes \Pi_{B_j}^{b_n} \otimes \Pi_{C_k}^{c_o}), \quad (10)$$

where $\Pi_{A_i}^{a_m} = 1/2 [\mathbb{1} + a_m \hat{a}_i \cdot \vec{\sigma}]$, $\Pi_{B_j}^{b_n} = 1/2 [\mathbb{1} + b_n \hat{b}_j \cdot \vec{\sigma}]$ and $\Pi_{C_k}^{c_o} = 1/2 [\mathbb{1} + c_o \hat{c}_k \cdot \vec{\sigma}]$ are the projectors generating binary outcomes $a_m, b_n, c_o \in \{-1, 1\}$. Any such tripartite quantum correlation can be written as a convex mixture of the extremal boxes of the tripartite NS polytope defined above. We refer to a set of projective measurements that give rise to the given quantum correlation as incompatible if the observables on each side is noncommuting, i.e., $[A_0, A_1] \neq 0$, $[B_0, B_1] \neq 0$ and $[C_0, C_1] \neq 0$.

III. SVETLICHNY-BOX POLYTOPE AND TWO NOTIONS OF GENUINE NONCLASSICALITY FOR SVETLICHNY-LOCAL BOXES

Svetlichny-box polytope, \mathcal{R} , is a restricted NS polytope in which we discard in total $53856 - 128 = 53728$ extremal

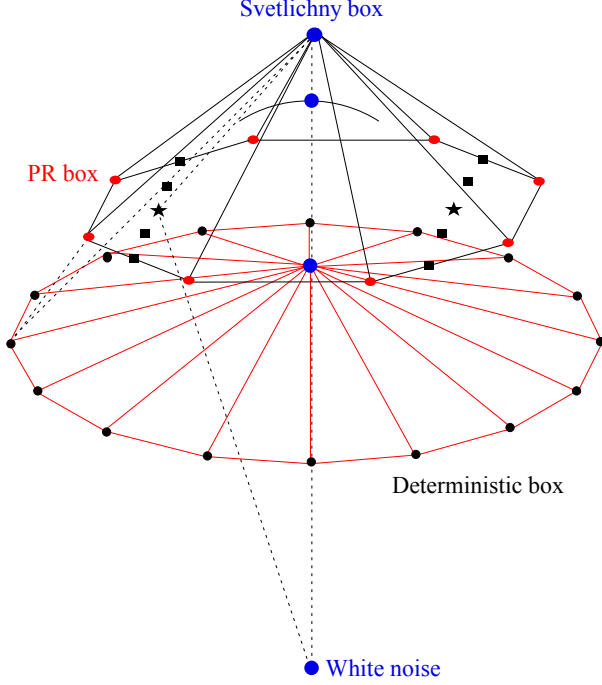


FIG. 1. A three-dimensional representation of the Svetlichny-box polytope is shown here. The fully deterministic boxes are represented by the circular points on the hexadecagon. The bipartite PR-boxes are represented by the circular points on the octagon. The circular point on the top represents the Svetlichny-box. The region that lies above the hexadecagon and below the octagon represents the two-way nonlocal region. The region below the curved surface contains quantum correlations and the point on this curved surface represents the quantum box that achieves maximal Svetlichny nonlocality. The star and square points represent Mermin boxes with maximally and nonmaximally mixed bipartite marginals, respectively. The triangular region (shown by dotted lines) which is a convex hull of the Svetlichny-box, the Mermin box and white noise represents the 3-decomposition fact that any point that lies inside the triangle can be decomposed into Svetlichny-box, the Mermin-box and white noise. The circular point at the center of the hexadecagon is the isotropic Svetlichny-box with $p_{Sv} = \frac{1}{2}$ which can be decomposed as an equal mixture of the 16 deterministic boxes or an equal mixture of two Mermin boxes.

boxes. The 128 extremal boxes of \mathcal{R} are the Svetlichny-boxes, the bipartite PR-boxes and the deterministic boxes. Svetlichny-box polytope is convex, i.e., if $P \in \mathcal{R}$,

$$P = \sum_{i=0}^{15} p_i P_{Sv}^i + \sum_{i=0}^{15} q_i P_{12}^i + \sum_{i=0}^{15} r_i P_{13}^i + \sum_{i=0}^{15} s_i P_{23}^i + \sum_{j=0}^{63} t_j P_D^j, \quad (11)$$

with $\sum_i p_i + \sum_i q_i + \sum_i r_i + \sum_i s_i + \sum_j t_j = 1$, $i = \alpha\beta\gamma\epsilon$ and $j = \alpha\beta\gamma\epsilon\zeta\eta$. Svetlichny-box polytope can be divided into a three-way nonlocal region and the two-way local polytope (\mathcal{L}_2). We will define two notions of genuine nonclassicality and show that any correlation in \mathcal{R} admits a three-way decomposition.

A. Svetlichny discord

Consider isotropic Svetlichny-box which is a convex mixture of the Svetlichny-box and white noise,

$$P = p_{Sv} P_{Sv}^{0000} + (1 - p_{Sv}) P_N. \quad (12)$$

Here white noise P_N is the maximally mixed box, i.e., $P_N = 1/8$ for all i, j, k, m, n, o . The isotropic Svetlichny-box violates the Svetlichny inequality, i.e., $\mathcal{S}_{0000} = 8p_{Sv} > 4$ if $p_{Sv} > \frac{1}{2}$. Notice that even if the isotropic Svetlichny-box is local for $p_{Sv} \leq \frac{1}{2}$, it admits a decomposition with the nonzero Svetlichny-box component. We call such a single Svetlichny-box in the decomposition of any box (three-way nonlocal, or not) irreducible Svetlichny-box.

The isotropic Svetlichny-box which is quantum realizable if $p_{Sv} \leq \frac{1}{\sqrt{2}}$ illustrates the following observation.

Observation 1. *When a Svetlichny-local box arising from a given genuinely entangled state has an irreducible Svetlichny-box component, the box arises from incompatible measurements that give rise to Svetlichny nonlocality.*

Proof. For the incompatible measurements $A_0 = \sigma_x$, $A_1 = \sigma_y$, $B_0 = \sigma_x$, $B_1 = \sigma_y$ and $C_k = \frac{1}{\sqrt{2}}(\sigma_x - (-1)^k \sigma_y)$, the GHZ state,

$$|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle), \quad (13)$$

violates the Svetlichny inequality, $\mathcal{S}_{0000} \leq 4$, to its quantum bound of $4\sqrt{2}$. For this choice of measurements, the GHZ states,

$$|\psi_{GGHZ}\rangle = \cos\theta|000\rangle + \sin\theta|111\rangle; \quad 0 \leq \theta \leq \frac{\pi}{4}, \quad (14)$$

give rise to the isotropic Svetlichny-box in Eq. (12) with $p_{Sv} = \frac{\sin 2\theta}{\sqrt{2}}$. Thus, the nonzero irreducible Svetlichny-box component implies the presence of incompatible measurements and genuine entanglement even if the correlation is local. \square

The observation that Svetlichny-local quantum correlations that have an irreducible Svetlichny-box component can arise from incompatible measurements performed on the genuinely entangled states motivates to define a notion of genuine nonclassicality which we call Svetlichny discord.

Definition 1. *A quantum correlation arising from incompatible measurements performed on a given three-qubit state is said to have Svetlichny discord iff the correlation admits a decomposition with an irreducible Svetlichny-box component.*

Svetlichny discord is of course not equivalent to Svetlichny nonlocality since there are Svetlichny-local boxes that have an irreducible Svetlichny-box component; for instance, the isotropic Svetlichny-box in Eq. (12) has Svetlichny discord if $p_{Sv} > 0$ and exhibits Svetlichny nonlocality if $p_{Sv} > \frac{1}{2}$.

We now define a measure of Svetlichny discord to detect irreducible Svetlichny-box component in any box by using the modulus of the Svetlichny functions in Eq. (9),

$$S_{\alpha\beta\gamma} = \left| \sum_{ijk} (-1)^{i \oplus j \oplus k} \langle A_i B_j C_k \rangle \right|. \quad (15)$$

Definition 2. Svetlichny discord, \mathcal{G} , is defined as,

$$\mathcal{G} = \min\{\mathcal{G}_1, \dots, \mathcal{G}_9\}, \quad (16)$$

where

$$\mathcal{G}_1 = \left| |S_{000} - S_{001}| - |S_{010} - S_{011}| \right| \\ - \left| |S_{100} - S_{101}| - |S_{110} - S_{111}| \right|,$$

and the other eight \mathcal{G}_i are obtained by permuting $S_{\alpha\beta\gamma}$ in \mathcal{G}_1 . Here $0 \leq \mathcal{G} \leq 8$.

Svetlichny discord is constructed such that it satisfies the following properties: (i) positivity, i.e., $\mathcal{G} \geq 0$, (ii) the bipartite PR-boxes and the deterministic boxes have $\mathcal{G} = 0$, (iii) the algebraic maximum of Svetlichny discord is achieved by the Svetlichny boxes, i.e., $\mathcal{G} = 8$ for any Svetlichny-box. Svetlichny discord is clearly invariant under LRO since the set $\{\mathcal{G}_i\}$ is invariant under LRO. Svetlichny discord divides the boxes in the two-way local polytope into two disjoint sets: $\mathcal{G} > 0$ boxes and $\mathcal{G} = 0$ boxes. Before characterizing the set of $\mathcal{G} > 0$ boxes, we make the following two observations.

Observation 2. The set of $\mathcal{G} = 0$ boxes forms a subpolytope of the two-way local polytope and is nonconvex.

Proof. The set of $\mathcal{G} = 0$ boxes is nonconvex since certain convex mixture of the $\mathcal{G} = 0$ boxes can have $\mathcal{G} > 0$; for instance, the isotropic Svetlichny-box in Eq. (12) can be written as the convex mixture of the deterministic boxes if $p_{SV} \leq \frac{1}{2}$, however, it has Svetlichny discord $\mathcal{G} = 8p_{SV} > 0$ if $p_{SV} > 0$. Thus, the set of $\mathcal{G} = 0$ boxes forms a nonconvex subpolytope of the two-way local polytope as the deterministic boxes and the bipartite PR-boxes have $\mathcal{G} = 0$. \square

Observation 3. An unequal mixture of any two Svetlichny-boxes: $pP_{SV}^i + qP_{SV}^j$, here $p > q$, can be written as the mixture of an irreducible Svetlichny-box and a Svetlichny-local box.

Proof. $pP_{SV}^i + qP_{SV}^j = (p - q)P_{SV}^i + 2qP_{SV}^{ij}$. Here $P_{SV}^{ij} = \frac{1}{2}(P_{SV}^i + P_{SV}^j)$ is a Svetlichny-local box since uniform mixture of any two Svetlichny-boxes belongs to the two-way local polytope. Notice that the second Svetlichny-box, P_{SV}^j , in the unequal mixture is not irreducible as its presence vanishes with the first Svetlichny-box in the other possible decomposition by the uniform mixture. \square

We obtain the following canonical decomposition of the boxes in \mathcal{R} .

Lemma 1. Any box that belongs to the Svetlichny-box polytope can be written as a convex mixture of an irreducible Svetlichny-box and a Svetlichny-local box with $\mathcal{G} = 0$,

$$P = \mu P_{SV}^{\alpha\beta\gamma\epsilon} + (1 - \mu) P_{SVL}^{\mathcal{G}=0}. \quad (17)$$

Proof. We now rewrite the decomposition of any box in \mathcal{R} given in Eq. (11) as the convex combination of the 16 Svetlichny-boxes and a Svetlichny-local box that does not have the Svetlichny-box components,

$$P = \sum_{i=0}^{15} g_i P_{SV}^i + \left(1 - \sum_{i=0}^{15} g_i \right) P_{SVL}, \quad (18)$$

here $P_{SVL} \neq \sum_{i=0}^{15} p'_i P_{SV}^i + \sum_{i=0}^{15} q'_i P_{12}^i + \sum_{i=0}^{15} r'_i P_{13}^i + \sum_{i=0}^{15} s'_i P_{23}^i + \sum_{j=0}^{63} t'_j P_D^j$, i.e., P_{SVL} cannot have nonzero p'_i . Thus the above decomposition is obtained by maximizing the Svetlichny-box components p_i in Eq. (11) overall possible decompositions. It follows from the observation 3 that the mixture of the Svetlichny-boxes in the first term in Eq. (18) can be written as a mixture of a single Svetlichny-box and the 15 Svetlichny-local boxes, P_{SVL}^i , which are the uniform mixture of two Svetlichny-boxes. The largest component of the Svetlichny-box which is unequal to any other Svetlichny-box components in Eq. (18) gives rise to irreducible Svetlichny-box component, μ :

$$\sum_i g_i P_{SV}^i = \mu P_{SV}^{\alpha\beta\gamma\epsilon} + \sum_{i=1}^{15} p_i P_{SVL}^i. \quad (19)$$

Here μ is obtained by minimizing the single Svetlichny-box excess overall possible decompositions such that the left hand side $\sum_i g_i P_{SV}^i \neq \sum_{i=1}^{15} q_i P_{SVL}^i$ and the second term in the right hand side $\sum_{i=1}^{15} p_i P_{SVL}^i$ has $\mathcal{G} = 0$. Substituting Eq. (19) in Eq. (18), we get the canonical decomposition for any box in \mathcal{R} ,

$$P = \mu P_{SV}^{\alpha\beta\gamma\epsilon} + (1 - \mu) P_{SVL}^{\mathcal{G}=0}, \quad (20)$$

where $P_{SVL}^{\mathcal{G}=0} = \frac{1}{1-\mu} \left\{ \sum_i p_i P_{SVL}^i + (1 - \sum_i g_i) P_{SVL} \right\}$. The fact that the Svetlichny-local box, $P_{SVL}^{\mathcal{G}=0}$, in this decomposition has $\mathcal{G} = 0$ follows from the geometry of the convex polytope that any point in the polytope lies along a line joining the two points of the polytope. Notice that \mathcal{G} divides the two-way local polytope into a $\mathcal{G} > 0$ region and $\mathcal{G} = 0$ polytope. Since the box in the first term in the decomposition given in Eq. (20) is from the $\mathcal{G} > 0$ region and the decomposition is for any box, the box in the second term must be from the $\mathcal{G} = 0$ polytope. \square

It follows from the canonical decomposition in Eq. (17) that a Svetlichny-local box has nonzero Svetlichny discord iff it admits a decomposition with an irreducible Svetlichny-box component.

Corrolary 1. Any box given by the decomposition in Eq. (17) has Svetlichny discord $\mathcal{G} = 8\mu$.

Proof. The nonextremal boxes in the two-way local polytope can have the following three types of decompositions due to the convexity of \mathcal{R} : (i) a convex mixture of two $\mathcal{G} = 0$ boxes, (ii) a convex mixture of two $\mathcal{G} > 0$ boxes and (iii) a convex mixture of a $\mathcal{G} > 0$ box and a $\mathcal{G} = 0$ box. Since certain convex mixture of the $\mathcal{G} = 0$ boxes ($\mathcal{G} > 0$ boxes) can have

$\mathcal{G} > 0$ ($\mathcal{G} = 0$), \mathcal{G} is, in general, not linear for the two decompositions (i) and (ii). However, \mathcal{G} is linear for the decomposition (iii) which implies that Svetlichny discord of the box given by the decomposition in Eq. (17) can be evaluated as, $\mathcal{G}(P) = \mu \mathcal{G}(P_{S_V}^{\alpha\beta\gamma\epsilon}) + (1 - \mu) \mathcal{G}(P_{S_V}^{\mathcal{G}=0}) = 8\mu > 0$ if $\mu > 0$. \square

Thus, we say that the decomposition of the boxes given in Eq. (17) is canonical in that it classifies any box in \mathcal{R} according to whether it has Svetlichny discord or not.

Corollary 2. *Irreducible Svetlichny-box component, μ , in the canonical decomposition given in Eq. (17) is invariant under LRO and permutations of the parties.*

Proof. Since \mathcal{G} is invariant under LRO and permutations of the parties, the irreducible Svetlichny-box component, μ , in Eq. (17) is invariant under LRO. \square

B. Mermin-boxes

Greenberger-Horne-Zeilinger (GHZ) paradox demonstrates multipartite nonlocality of the maximally genuine multipartite entangled states without using any statistical inequality [18]. For the following choice of incompatible measurements: $A_0 = \sigma_x$, $A_1 = \sigma_y$, $B_0 = \sigma_x$, $B_1 = \sigma_y$, $C_0 = \sigma_x$, and $C_1 = \sigma_y$, the correlation arising from the maximally entangled three-qubit state, $|\psi_{GHZ}\rangle$, exhibits the GHZ paradox [19]. For the above measurements on $|\psi_{GHZ}\rangle$, the outcomes satisfy the following relation:

$$A_0 B_0 C_0 = -A_0 B_1 C_1 = -A_1 B_0 C_1 = -A_1 B_1 C_0 = 1. \quad (21)$$

It can be inferred from this relation that the correlation exhibits logical contradiction with local-realistic value assignment to the observables. However, this correlation does not exhibit genuine three-way nonlocality as it can be decomposed into an equal mixture of the four bipartite PR-boxes as follows,

$$P_M(a_m, b_n, c_o | A_i, B_j, C_k) = \frac{1}{4} \sum_{\lambda=1}^4 P_\lambda(a_m | A_i) P_\lambda(b_n, c_o | B_j, C_k), \quad (22)$$

where $P_1(a_m | A_i) = \delta_{m \oplus i}^i$, $P_2(a_m | A_i) = \delta_{m \oplus i \oplus 1}^i$, $P_3(a_m | A_i) = \delta_{m \oplus 1}^i$, $P_4(a_m | A_i) = \delta_m^i$, $P_1(b_n, c_o | B_j, C_k) = P_{PR}^{110}$, $P_2(b_n, c_o | B_j, C_k) = P_{PR}^{111}$, $P_3(b_n, c_o | B_j, C_k) = P_{PR}^{001}$ and $P_4(b_n, c_o | B_j, C_k) = P_{PR}^{000}$. Thus, the correlation which exhibits the GHZ paradox belongs to the nonlocal region of the two-way local polytope. Further, the correlation in Eq. (22) can also arise from a non-genuinely entangled state in the higher dimensional Hilbert space (see Appendix A).

The logical contradiction of the type given in Eq. (21) can be captured by the maximal violation of a Mermin inequality [15]. There are 16 Mermin inequalities which are invariant under LRO and are given as follows [20],

$$\mathcal{M}_{\alpha\beta\gamma\epsilon} = (\alpha \oplus \beta \oplus \gamma \oplus 1) \mathcal{M}_{\alpha\beta\gamma\epsilon}^+ + (\alpha \oplus \beta \oplus \gamma) \mathcal{M}_{\alpha\beta\gamma\epsilon}^- \leq 2, \quad (23)$$

where

$$\begin{aligned} \mathcal{M}_{\alpha\beta\gamma\epsilon}^+ &:= (-1)^{\gamma \oplus \epsilon} \langle A_0 B_0 C_1 \rangle + (-1)^{\beta \oplus \epsilon} \langle A_0 B_1 C_0 \rangle \\ &\quad + (-1)^{\alpha \oplus \epsilon} \langle A_1 B_0 C_0 \rangle + (-1)^{\alpha \oplus \beta \oplus \gamma \oplus \epsilon \oplus 1} \langle A_1 B_1 C_1 \rangle \\ \mathcal{M}_{\alpha\beta\gamma\epsilon}^- &:= (-1)^{\alpha \oplus \beta \oplus \epsilon \oplus 1} \langle A_1 B_1 C_0 \rangle + (-1)^{\alpha \oplus \gamma \oplus \epsilon \oplus 1} \langle A_1 B_0 C_1 \rangle \\ &\quad + (-1)^{\beta \oplus \gamma \oplus \epsilon \oplus 1} \langle A_0 B_1 C_1 \rangle + (-1)^\epsilon \langle A_0 B_0 C_0 \rangle. \end{aligned}$$

The Mermin inequalities serve as the criterion for the tripartite EPR-steering under the constraint that the measurements chosen by each party is noncommuting [21]. In the seminal paper, Mermin inequality was derived by using anti-commuting observable on each side to show that the correlations arising from the genuinely multipartite entangled states are incompatible with the fully separable LHV model [15]. Furthermore, this Mermin inequality is equivalent to a noncontextual inequality [22].

There are two types of two-way nonlocal boxes which can be distinguished according to whether nonlocality is due to the tripartite correlation or the marginal correlation between any two parties. The correlations which exhibit the GHZ paradox can also be decomposed into the uniform mixture of two particular Svetlichny-boxes; for instance, the correlation in Eq. (22) can also be decomposed as,

$$P_M = \frac{1}{2} (P_{S_V}^{0000} + P_{S_V}^{1110}). \quad (24)$$

Thus, the nonlocality of these maximally two-way nonlocal boxes is not due to any of the bipartite marginal correlations as they have maximally mixed bipartite marginals. The maximally two-way nonlocal boxes are the ones which maximally violate a Mermin inequality. Notice that two Mermin inequalities are maximally violated by any bipartite PR-box.

Definition 3. *We call a maximally two-way nonlocal box Mermin box if it violates only one of the Mermin inequalities.*

Just like there are 16 Svetlichny-boxes maximally violating only one of the Svetlichny inequalities, there are 16 tripartite Mermin-boxes arising from the GHZ states which maximally violate only one of the Mermin inequalities given in Eq. (23).

Mermin illustrated that the measurements associated with the GHZ paradox exhibits KS paradox that illustrates contextuality in addition to Bell nonlocality [19]. The observation of the GHZ paradox using the incompatible measurements implies the presence of genuine entanglement.

Definition 4. *A two-way nonlocal quantum correlation is said to have three-way contextuality if it arises from a genuinely entangled state.*

In analogy with Svetlichny-boxes which exhibit maximal three-way nonlocality, we say that a Mermin-box which is the uniform mixture of two Svetlichny boxes is maximally three-way contextual when it exhibits the GHZ paradox. Not all uniform mixture of two Svetlichny-boxes can have three-way contextuality; for instance, white noise can be decomposed into the uniform mixture of the two Svetlichny-boxes. The uniform mixture of two Svetlichny-boxes in a Mermin-box

which exhibits the GHZ paradox destroys three-way nonlocality; however, the perfect correlations left in it for the four joint measurements, $A_i B_j C_k$, leads to genuine three-way contextuality [19]. Notice that if we permute the party's indices in the decomposition in Eq. (22), the Mermin box remains invariant. Thus, three-way contextuality of the correlations is symmetric under the permutations of the parties.

Two-way local polytope admits two types of Mermin-boxes which can be distinguished by their marginals.

Observation 4. *The nonmaximally mixed bipartite marginal Mermin-boxes cannot have three-way contextuality.*

Proof. Consider the following uniform mixture of two bipartite PR-boxes,

$$P = \frac{1}{2} \sum_{\lambda=1}^2 P_{\lambda}(a_m|A_i) P_{\lambda}(b_n, c_o|B_j, C_k) \quad (25)$$

where $P_1(a_m|A_i) = \delta_{m \oplus i}^i$, $P_2(a_m|A_i) = \delta_{m \oplus 1}^i$, $P_1(b_n, c_o|B_j, C_k) = P_{PR}^{110}$, and $P_2(b_n, c_o|B_j, C_k) = P_{PR}^{001}$. Notice that this correlation which has nonmaximally mixed marginals and the Mermin box in Eq. (22) which has maximally mixed marginals are equivalent with respect to the joint expectations $\langle A_i B_j C_k \rangle$. Thus, the correlation in Eq. (25) also maximally violate only one of the Mermin inequalities. Since the nonlocality of the box in Eq. (25) is not due to a bipartite marginal correlation, it also represents a Mermin box, but with nonmaximally mixed marginal. Notice that if we permute the party's indices in the decomposition in Eq. (25), it will give rise to the another Mermin-box. This indicates that it does not have genuine three-way contextuality. Indeed, the Mermin boxes with a nonmaximally mixed bipartite marginal do not arise from a genuinely entangled state (see Appendix A). \square

C. Mermin discord and 3-decomposition

Consider isotropic Mermin-box which is a convex mixture of the Mermin-box in Eq. (22) and white noise,

$$P = p_M P_M + (1 - p_M) P_N, \quad (26)$$

The isotropic Mermin-box violates the Mermin inequality i.e., $\mathcal{M}_{0010} = 4p_M > 2$ if $p_M > \frac{1}{2}$. Notice that even if the isotropic Mermin-box is local for $p_M \leq \frac{1}{2}$, it admits a decomposition with the nonzero Mermin-box component. We call such a single Mermin-box in the decomposition of any box (nonlocal, or not) irreducible Mermin-box.

The following observation can be illustrated by the isotropic Mermin-box.

Observation 5. *When a local quantum correlation arising from a given genuinely entangled state has an irreducible Mermin-box component, the correlation arises from incompatible measurements that give rise to three-way contextuality.*

Proof. For the incompatible measurements that give rise to the GHZ paradox in Eq. (21), the GGHZ states in Eq. (14) give rise to the isotropic Mermin-box in Eq. (26) with $p_M = \sin 2\theta$. Thus, the nonzero irreducible Mermin-box component implies the presence of incompatible measurements and genuine entanglement even if the correlation is local. \square

The observation that local quantum correlations that have an irreducible Mermin-box component can arise from incompatible measurements performed on the genuinely entangled states motivates to define a notion of genuine nonclassicality which we call Mermin discord.

Definition 5. *A quantum correlation arising from incompatible measurements performed on a given three-qubit state is said to have Mermin discord iff the correlation admits a decomposition with an irreducible Mermin-box component.*

Mermin discord is not equivalent to three-way contextuality since the boxes that do not violate a Mermin inequality can also have an irreducible Mermin-box component; for instance, the isotropic Mermin-box in Eq. (26) which can exhibit three-way contextuality if $p_M > \frac{1}{2}$ has Mermin discord if $p_M > 0$.

Observation 6. *For any Mermin-box, only one of the Mermin functions, $\mathcal{M}_{\alpha\beta\gamma} := |\mathcal{M}_{\alpha\beta\gamma\epsilon}|$, attains the maximum and the rest of them take zero, where $\mathcal{M}_{\alpha\beta\gamma\epsilon}$ are the Mermin operators given in Eq. (23).*

The above observation motivates us to define a measure of Mermin discord using the Mermin functions, similar to the measure of Svetlichny discord.

Definition 6. *Mermin discord, Q , is defined as,*

$$Q = \min\{Q_1, \dots, Q_9\}, \quad (27)$$

where

$$Q_1 = \left| |\mathcal{M}_{000} - \mathcal{M}_{001}| - |\mathcal{M}_{010} - \mathcal{M}_{011}| \right| - \left| |\mathcal{M}_{100} - \mathcal{M}_{101}| - |\mathcal{M}_{110} - \mathcal{M}_{111}| \right|,$$

and the other eight Q_i are obtained by permuting $\mathcal{M}_{\alpha\beta\gamma}$ in Q_1 . Here $0 \leq Q \leq 4$.

Mermin discord is constructed such that it satisfies the following properties: (i) $Q = 0$ for the Svetlichny-boxes, bipartite PR-boxes and deterministic boxes (ii) the algebraic maximum of Q is achieved by the Mermin boxes, i.e., $Q = 4$ for any Mermin-box and (iii) Q is invariant under LRO since the set $\{Q_i\}$ is invariant under LRO.

We obtain the following observations from the Mermin discord defined in Eq. (27).

Observation 7. *The set of $Q = 0$ boxes in \mathcal{R} forms a nonconvex subpolytope of the full Svetlichny-box polytope.*

Proof. Since the extremal boxes of the Svetlichny-box polytope have $Q = 0$, and certain convex mixture of the $Q = 0$ boxes can have $Q > 0$, the set of $Q = 0$ boxes forms a nonconvex subpolytope of the full Svetlichny-box polytope. \square

Observation 8. \mathcal{Q} divides the $\mathcal{G} = 0$ polytope into a $\mathcal{Q} > 0$ region and $\mathcal{G} = \mathcal{Q} = 0$ nonconvex polytope.

Proof. Since all the bipartite PR-boxes and deterministic boxes have $\mathcal{G} = \mathcal{Q} = 0$ and certain convex mixture of these extremal boxes can have $\mathcal{Q} > 0$, the set of $\mathcal{G} = \mathcal{Q} = 0$ boxes forms a nonconvex subpolytope of the $\mathcal{G} = 0$ polytope. \square

Observation 9. A $\mathcal{Q} = 4$ box is, in general, a convex combination of a Mermin-box with maximally mixed marginals and the 12 nonmaximally mixed marginal Mermin-boxes which are equivalent with respect to $\langle A_i B_j C_k \rangle$,

$$P_{\mathcal{Q}=4} = u P_M^{\mathcal{Q}} + \sum_{i=1}^{12} v_i P_M^{n\mathcal{Q}_i}, \quad (28)$$

where $P_M^{\mathcal{Q}}$ has maximally mixed bipartite marginals and $P_M^{n\mathcal{Q}_i}$ have a nonmaximally mixed bipartite marginal; all the Mermin-boxes in this decomposition violate the same Mermin inequality as they are equivalent with respect to $\langle A_i B_j C_k \rangle$.

Proof. Notice that any convex mixture of the two Mermin boxes in Eqs. (22) and (25) have $\mathcal{Q} = 4$. It can be checked that there can be four Mermin boxes which are the uniform mixture of two bipartite PR-boxes in Eq. (22). The permutation of the party's indices in these boxes give rise to the another 8 Mermin boxes. Since these 12 Mermin boxes are equivalent with respect to $\langle A_i B_j C_k \rangle$ corresponding to a given Mermin box which has maximally mixed bipartite marginals, any convex mixture of these 13 Mermin boxes have $\mathcal{Q} = 4$. \square

We obtain the following 3-decomposition fact of the Svetlichny-box polytope.

Theorem 1. Any box in \mathcal{R} given by the decomposition in Eq. (11) can be written as a convex mixture of a Svetlichny-box, a maximally two-way nonlocal box with $\mathcal{Q} = 4$ and a box with $\mathcal{G} = \mathcal{Q} = 0$,

$$P = \mu P_{Sv}^{\alpha\beta\gamma\epsilon} + \nu P_{\mathcal{Q}=4} + (1 - \mu - \nu) P_{\mathcal{Q}=0}^{\mathcal{G}=0}. \quad (29)$$

Proof. Since all the Mermin-boxes have $\mathcal{G} = 0$, they belong to the $\mathcal{G} = 0$ polytope. This implies that any $\mathcal{G} = 0$ box can be written as a convex mixture of the Mermin-boxes and a Svetlichny-local box that does not have the Mermin-box components,

$$P_{SvL}^{\mathcal{G}=0} = \sum_{i=0}^{15} u_i P_M^{\mathcal{Q}_i} + \sum_{j=1}^{192} v_j P_M^{n\mathcal{Q}_j} + \left(1 - \sum_{i=0}^{15} u_i - \sum_{j=1}^{192} v_j\right) P_{SvL}, \quad (30)$$

where $P_M^{\mathcal{Q}_i}$ and $P_M^{n\mathcal{Q}_j}$ are the Mermin-boxes with maximally mixed marginals and a nonmaximally mixed bipartite marginal, respectively. It follows from the observation 9 that the mixture of the Mermin boxes in this decomposition can be written as the mixture of the 16 maximally two-way nonlocal boxes with $\mathcal{Q} = 4$. Notice that unequal mixture of any two $\mathcal{Q} = 4$ boxes that violate the two different Mermin inequalities in Eq. (23): $p P_{\mathcal{Q}=4}^1 + q P_{\mathcal{Q}=4}^2$, $p > q$, can be written as a mixture of an irreducible $\mathcal{Q} = 4$ box and a local box which is a uniform

mixture of the two $\mathcal{Q} = 4$ boxes: $(p - q) P_{\mathcal{Q}=4}^1 + 2q P_L$, here $P_L = \frac{1}{2} (P_{\mathcal{Q}=4}^1 + P_{\mathcal{Q}=4}^2)$ is a Bell-local box which has $\mathcal{Q} = 0$. Therefore, the sum of the first two terms in the decomposition given in Eq. (30) can be written as a mixture of an irreducible $\mathcal{Q} = 4$ box and a Bell-local box,

$$\sum_{i=0}^{15} u_i P_M^{\mathcal{Q}_i} + \sum_j v_j P_M^{n\mathcal{Q}_j} = \zeta P_{\mathcal{Q}=4} + \sum_{i=1}^{15} l_i P_L^i, \quad (31)$$

where P_L^i are the Bell-local boxes which are the uniform mixture of two $\mathcal{Q} = 4$ boxes. Here ζ is obtained by minimizing the single $\mathcal{Q} = 4$ box excess overall possible decompositions such that the left hand side $\sum_{i=0}^{15} u_i P_M^{\mathcal{Q}_i} + \sum_j v_j P_M^{n\mathcal{Q}_j} \neq \sum_{i=1}^{15} l_i P_L^i$ and the second term in the right hand side $\sum_{i=1}^{15} l_i P_L^i$ has $\mathcal{Q} = 0$. Substituting Eq. (31) in Eq. (30), we obtain the canonical decomposition for the $\mathcal{G} = 0$ boxes,

$$P_{SvL}^{\mathcal{G}=0} = \zeta P_{\mathcal{Q}=4} + (1 - \zeta) P_{\mathcal{Q}=0}^{\mathcal{G}=0}, \quad (32)$$

where $P_{\mathcal{Q}=0}^{\mathcal{G}=0} = \frac{1}{1-\zeta} \left\{ \sum_{i=1}^{15} l_i P_L^i + \left(1 - \sum_{i=0}^{15} u_i - \sum_j v_j\right) P_{SvL} \right\}$. The fact that the box in the second term in this decomposition has $\mathcal{G} = \mathcal{Q} = 0$ follows from the geometry of the $\mathcal{G} = 0$ polytope: The observation 8 implies that any box in the $\mathcal{G} = 0$ polytope lies on a line segment joining a $\mathcal{Q} > 0$ box and a $\mathcal{G} = \mathcal{Q} = 0$ box. This implies that the box in the second term in the decomposition given in Eq. (32) must have $\mathcal{G} = \mathcal{Q} = 0$ as the box in the first term has $\mathcal{Q} > 0$. Thus, decomposing the $\mathcal{G} = 0$ box in Eq. (17) as given in Eq. (32) gives the canonical decomposition given in Eq. (29) with $\nu = \zeta(1 - \mu)$. \square

Corrolary 3. A box has nonzero Mermin discord iff it admits a decomposition with an irreducible Mermin box component since Mermin discord $\mathcal{Q} = 4\nu$ for the box given by the canonical decomposition in Eq. (29).

Proof. Any box in \mathcal{R} given by the 3-decomposition in Eq. (29) can be written as a convex mixture of a maximally two-way nonlocal box with $\mathcal{Q} = 4$ and a box with $\mathcal{Q} = 0$,

$$P = \nu P_{\mathcal{Q}=4} + (1 - \nu) P_{\mathcal{Q}=0}, \quad (33)$$

where $P_{\mathcal{Q}=0} = \frac{1}{1-\nu} \left((1 - \mu - \nu) P_{\mathcal{Q}=0}^{\mathcal{G}=0} + \mu P_{Sv}^{\alpha\beta\gamma\epsilon} \right)$. The nonconvexity property of the $\mathcal{Q} = 0$ polytope implies that certain convex combination of the $\mathcal{Q} = 0$ boxes can have $\mathcal{Q} > 0$ and there are $\mathcal{Q} = 0$ boxes which can be written as a convex mixture of two $\mathcal{Q} > 0$ boxes. Thus, \mathcal{Q} is not linear for these two types of decomposition. However, \mathcal{Q} is linear for the decomposition given in Eq. (33) since the convex mixture of a $\mathcal{Q} > 0$ box and a $\mathcal{Q} = 0$ box is always a $\mathcal{Q} > 0$ box. Therefore, Mermin discord of the box in Eq. (33) is given by $\mathcal{Q}(P) = \nu \mathcal{Q}(P_{\mathcal{Q}=4}) + (1 - \nu) \mathcal{Q}(P_{\mathcal{Q}=0}^{\mathcal{G}=0}) = 4\nu > 0$ if $\nu > 0$. As any box that has an irreducible Mermin-box component lies on a line segment joining a Mermin-box and a $\mathcal{Q} = 0$ box, it has $\mathcal{Q} > 0$. \square

D. Monogamy between the measures

As the total amount of Svetlichny-box and Mermin-box components of any box given by the 3-decomposition in Eq.

(29) is constrained, i.e., $\mu + \nu \leq 1$, we obtain the following trade-off relation.

Corrolory 4. *Svetlichny and Mermin discord of any given box satisfy the following monogamy relation,*

$$\mathcal{G} + 2Q \leq 8. \quad (34)$$

This tradeoff relation reveals monogamy between three-way contextuality and three-way nonlocality of quantum correlations and is analogous to the monogamy relations between locally contextual correlations and nonlocal correlations derived by Kurzyński *et al.* [23]. The monogamy relations given by Kurzyński *et al.* implies that when measurements on qutrit system gives rise to contextuality in a qutrit-qubit entangled system, then these measurements do not give rise to nonlocality for all measurements on qubit system. Similar monogamy character follows from the observations 1 and 5: For the measurements that gives rise to the GHZ paradox, the GHZ state gives rise to maximal Mermin discord and zero Svetlichny discord, i.e., $Q = 4$ and $\mathcal{G} = 0$ which is consistent with Eq. (34). Thus, for the measurements that give rise to the GHZ paradox, the GGHZ states give rise to only Mermin discord i.e., $Q = 4 \sin 2\theta$ and $\mathcal{G} = 0$. Notice that for the measurements that gives rise to maximal three-way nonlocality, the GGHZ states give rise to only Svetlichny discord, i.e., $\mathcal{G} = 4\sqrt{2} \sin 2\theta$ and $Q = 0$. Thus, we see that the measurements that gives rise to extremal three-way contextuality do not give rise to three-way nonlocality and vice versa.

For general incompatible measurements, quantum correlations can have three-way contextuality and three-way nonlocality simultaneously, however, the tradeoff exists between these two forms of genuine nonclassicality as given by Eq. (34). For instance, the correlations arising from the GHZ state for the measurements $A_0 = \sigma_x$, $A_1 = \sigma_y$, $B_0 = \sqrt{p}\sigma_x - \sqrt{1-p}\sigma_y$, $B_1 = \sqrt{1-p}\sigma_x + \sqrt{p}\sigma_y$, $C_0 = \sigma_x$ and $C_1 = \sigma_y$, here $\frac{1}{2} \leq p \leq 1$, can be decomposed into the Svetlichny-box, the Mermin-box which is a uniform mixture of two Svetlichny-boxes, and white noise as follows,

$$P = \mu P_{Sv}^{0000} + \nu \left(\frac{P_{Sv}^{0000} + P_{Sv}^{1110}}{2} \right) + (1 - \mu - \nu) P_N, \quad (35)$$

where $\mu = \sqrt{1-p}$, $\nu = \sqrt{p} - \sqrt{1-p}$. These correlations have $\mathcal{G} + Q = 4\sqrt{p} \leq 4$.

IV. QUANTUM CORRELATIONS

The 3-decomposition obtained in Eq. (29) and the observation that the Mermin-boxes with a nonmaximally mixed marginal cannot be produced by a three-qubit state imply that any three-qubit correlation in the Svetlichny-box polytope can be decomposed into Svetlichny-box, a Mermin box with maximally mixed marginals and a box with $\mathcal{G} = Q = 0$,

$$P = \mu P_{Sv}^{\alpha\beta\gamma\epsilon} + \nu P_M + (1 - \mu - \nu) P_{Q=0}^{\mathcal{G}=0}. \quad (36)$$

We will characterize genuine nonclassicality of quantum correlations arising from local projective measurements along the

directions \hat{a}_i , \hat{b}_j and \hat{c}_k on the three-qubit systems using this three-way decomposition.

We will apply Svetlichny and Mermin discord to two inequivalent classes of pure genuinely entangled states [24] and the noisy GHZ state. For these states, a nonzero Svetlichny discord originates from incompatible measurements that give rise to Svetlichny nonlocality. Similarly, a nonzero Mermin discord originates from incompatible measurements that give rise to three-way contextuality. For a given genuinely nonclassical quantum state, there are three different incompatible measurements corresponding to (i) Svetlichny discordant correlation which has $\mathcal{G} > 0$ and $Q = 0$, (ii) Mermin discordant correlation which has $\mathcal{G} = 0$ and $Q > 0$ and (iii) Svetlichny-Mermin discordant correlation which has $\mathcal{G} > 0$ and $Q > 0$. Three-way nonlocal quantum correlations in \mathcal{R} are the subset of $\mathcal{G} > 0$ correlations, whereas three-way contextual correlations are the subset of $Q > 0$ correlations.

Svetlichny (Mermin) discord for a given nonclassical state is maximized by minimizing the number of nonzero Svetlichny (Mermin) functions overall incompatible measurements that give rise to $\mathcal{G} > 0$ ($Q > 0$). In the subsequent sections, we will choose the following four measurement settings:

$$\hat{a}_0 = \hat{x}, \quad \hat{a}_1 = \hat{y}, \quad \hat{b}_j = \frac{1}{\sqrt{2}} (\hat{x} + (-1)^{j \oplus 1} \hat{y}), \quad \hat{c}_0 = \hat{x}, \quad \hat{c}_1 = \hat{y} \quad (37)$$

$$\hat{a}_0 = \hat{z}, \quad \hat{a}_1 = \hat{x}, \quad \hat{b}_j = \frac{1}{\sqrt{2}} (\hat{z} + (-1)^j \hat{x}), \quad \hat{c}_0 = \hat{z}, \quad \hat{c}_1 = \hat{x} \quad (38)$$

$$\hat{a}_0 = \hat{x}, \quad \hat{a}_1 = \hat{y}, \quad \hat{b}_0 = \hat{x}, \quad \hat{b}_1 = \hat{y}, \quad \hat{c}_0 = \hat{x}, \quad \hat{c}_1 = \hat{y} \quad (39)$$

$$\hat{a}_0 = \hat{z}, \quad \hat{a}_1 = \hat{x}, \quad \hat{b}_0 = \hat{z}, \quad \hat{b}_1 = \hat{x}, \quad \hat{c}_0 = \hat{z}, \quad \hat{c}_1 = \hat{x} \quad (40)$$

for studying correlations arising from the genuinely nonclassical quantum states. We will apply Svetlichny and Mermin discord to various states in order to illustrate the new insights that may be obtained regarding the origin of genuine nonclassicality.

A. GHZ-class states

The GHZ-class states which have bipartite entanglement between A and B are given as follows,

$$|\psi_{gs}\rangle = \cos \theta |000\rangle + \sin \theta |11\rangle \left\{ \cos \theta_3 |0\rangle + \sin \theta_3 |1\rangle \right\}. \quad (41)$$

The genuine tripartite entanglement is quantified by the three tangle [25], $\tau_3 = (\sin 2\theta \sin \theta_3)^2$, and the bipartite entanglement is quantified by the concurrence [26], $C_{12} = \sin 2\theta \cos \theta_3$.

1. Svetlichny discordant box

The settings in Eq. (37) maximizes Svetlichny discord for the GHZ-class states, since the correlations have only one of

the Svetlichny functions nonzero i.e., $S_{000} = 4\sqrt{2\tau_3}$ and the rest of the Svetlichny functions are zero which implies that Svetlichny discord $\mathcal{G} = 4\sqrt{2\tau_3}$. These correlations can be decomposed as follows,

$$P = \frac{\sqrt{\tau_3}}{\sqrt{2}} P_{Sv}^{0000} + \left(1 - \frac{\sqrt{\tau_3}}{\sqrt{2}}\right) P_{SvL}^{\mathcal{G}=0}, \quad (42)$$

where the $\mathcal{G} = 0$ box, $P_{SvL}^{\mathcal{G}=0}$, which has nonmaximally mixed marginals becomes white noise for the GHZ states. The above correlations are Svetlichny-local if $0 \leq \tau_3 \leq \frac{1}{2}$, however, they have genuine nonclassicality originating from incompatible measurements that give rise to Svetlichny nonlocality if $\tau_3 > 0$.

2. Mermin discordant box

The settings in Eq. (39) maximizes Mermin discord for the GHZ-class states, since the correlations have only one of the Mermin functions is nonzero. These correlations can be written as a convex mixture of the Mermin-box and a Bell-local box:

$$P = \sqrt{\tau_3} \left(\frac{P_{Sv}^{0000} + P_{Sv}^{1110}}{2} \right) + (1 - \sqrt{\tau_3}) P_L^{Q=0}, \quad (43)$$

where the Bell-local box, $P_L^{Q=0}$, which has $Q = 0$ and nonmaximally mixed marginals becomes white noise for the GHZ states. The above correlations have Mermin discord $Q = 4\sqrt{\tau_3}$. Despite the correlations violate the Mermin inequality only if $\tau_3 > \frac{1}{4}$, they have genuine three-way nonclassicality originating from three-way contextuality if $\tau_3 > 0$.

3. Svetlichny-Mermin discordant box

For the following state dependent measurement settings: $\hat{a}_0 = \hat{x}$, $\hat{a}_1 = \hat{y}$, $\hat{b}_0 = \sin 2\theta \hat{x} - \cos 2\theta \hat{y}$, $\hat{b}_1 = \cos 2\theta \hat{x} + \sin 2\theta \hat{y}$, $\hat{c}_0 = \hat{x}$ and $\hat{c}_1 = \hat{y}$, the correlations arising from the GHZ states in Eq. (14) have Svetlichny and Mermin discord simultaneously as follows:

$$\begin{aligned} \mathcal{G} &= \begin{cases} 8\tau_3 & \text{when } 0 \leq \theta \leq \frac{\pi}{8} \\ 8\sqrt{\tau_3(1-\tau_3)} & \text{when } \frac{\pi}{8} \leq \theta \leq \frac{\pi}{4} \end{cases} \\ &> 0 \quad \text{if } \tau_3 \neq 0, 1 \\ Q &= 4 \left| \tau_3 - \sqrt{\tau_3(1-\tau_3)} \right| \\ &> 0 \quad \text{if } \tau_3 \neq 0, \frac{1}{2}. \end{aligned}$$

These correlation have a 3-decomposition as follows,

$$P = \mu P_{Sv}^{0000} + \nu \left(\frac{P_{Sv}^{0000} + P_{Sv}^{111\gamma}}{2} \right) + (1 - \mu - \nu) P_N, \quad (44)$$

where $\mu = \mathcal{G}/8$ and $\nu = Q/4$. Since the measurement settings corresponds to the GHZ paradox when $\theta = \pi/4$ and maximal three-way nonlocality when $\theta = \pi/8$, the correlation has zero irreducible Svetlichny-box component when $\theta = \pi/4$ and zero irreducible Mermin-box component when $\theta = \pi/8$.

4. Svetlichny-box polytope vs three-way nonlocal quantum correlations

Bancal *et al.* [17] conjectured that all pure genuinely entangled states can give rise to three-way nonlocal correlations and it was noticed that there are three-way nonlocal quantum correlations arising from the pure states which do not violate a Svetlichny inequality. In Ref. [27], it has been shown that all the GHZ states can give rise to the violation of a class 99 facet inequality whose representative is given in Eq. (7). For instance, the correlations arising from the GHZ states in Eq. (14) have $\mathcal{L}_2^{99} = 1 + 2\sqrt{1 + \sin^2 2\theta} > 3$ if $\tau_3 > 0$ for the measurement settings $\hat{a}_0 = \hat{z}$, $\hat{a}_1 = \hat{x}$, $\hat{b}_j = \cos t \hat{z} + (-1)^j \sin t \hat{x}$, $\hat{c}_0 = \hat{z}$ and $\hat{c}_1 = \hat{x}$, where $\cos t = 1/\sqrt{1 + \sin^2 2\theta}$. For $\theta = \frac{\pi}{4}$, the correlation violates this inequality to its quantum bound of $1 + 2\sqrt{2}$ and can be decomposed in a convex mixture of the class 8 extremal box given in the table of Ref. [14] and a local box,

$$P = \frac{1}{\sqrt{2}} P_8 + \left(1 - \frac{1}{\sqrt{2}}\right) P_L. \quad (45)$$

Here P_L arises from the state $\rho = \rho_{AC} \otimes \frac{\mathbb{1}}{2}$, where $\rho_{AC} = \frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$. As genuine nonlocality of the correlation is due to the class 8 extremal box, it does not violate a Svetlichny inequality and hence it does not belong to the three-way nonlocal region of the Svetlichny-box polytope. Notice that the correlation in Eq. (45) has $\mathcal{G} = Q = 0$.

B. W-class states

We now study the correlations arising from the W-class states,

$$|\psi_w\rangle = \alpha|100\rangle + \beta|010\rangle + \gamma|001\rangle, \quad (46)$$

For these states, we may consider the three nonvanishing bipartite concurrences $C_{12} = 2\alpha\beta$, $C_{13} = 2\alpha\gamma$ and $C_{23} = 2\beta\gamma$ or the minimal concurrence of assistance [28] $C_{min}^a = \min\{C_{12}, C_{13}, C_{23}\}$ as genuine tripartite entanglement measure. The optimal settings that maximizes Svetlichny/Mermin discord for the GHZ-class states do not maximize Svetlichny/Mermin discord for the W-class states.

1. Svetlichny discordant box

Svetlichny discord for the W-class states is maximized by the settings in Eq. (38) which gives rise to,

$$\mathcal{G} = \min_{i=1}^3 \mathcal{G}_i = 4\sqrt{2}C_{min}^a > 0 \quad \text{iff } C_{12}C_{23} > 0,$$

where

$$\begin{aligned} \mathcal{G}_1 &= \sqrt{2} \left| |1 + C_{12} + C_{13} + C_{23}| - |1 + C_{12} - C_{13} - C_{23}| \right| \\ &\quad - \left| |1 - C_{12} - C_{13} + C_{23}| - |1 - C_{12} + C_{13} - C_{23}| \right|, \end{aligned}$$

and \mathcal{G}_2 and \mathcal{G}_3 are obtained by permuting the four $\mathcal{S}_{\alpha\beta\gamma}$ in \mathcal{G}_1 . The correlations can be decomposed in a convex mixture of a Svetlichny-box and a Svetlichny-local box which has $\mathcal{G} = 0$ as follows,

$$P = \frac{C_{\min}^a}{\sqrt{2}} P_{Sv}^{0100} + \left(1 - \frac{C_{\min}^a}{\sqrt{2}}\right) P_{SvL}^{\mathcal{G}=0}. \quad (47)$$

The above correlations do not violate a Svetlichny inequality when $C_{12} + C_{13} + C_{23} \leq 2\sqrt{2} - 1$, however, Svetlichny discord is nonzero whenever the state is genuinely entangled. The Svetlichny-local box in Eq. (47) must have a decomposition which has the class 8 extremal box as the correlations also violate a class 99 facet inequality of \mathcal{L}_2 when $C_{13} + \frac{1}{\sqrt{2}}(C_{12} + C_{23}) > 3 - \sqrt{2}$. Therefore, the three-way nonlocal correlations arising from the W-class states lie in the overlap between the Svetlichny-box polytope and full NS polytope.

2. Mermin discordant box

Mermin discord for the W-class states is maximized by settings in Eq. (40) which gives rises to,

$$Q = \min_{i=1}^3 Q_i = 4C_{\min}^a > 0 \quad \text{iff} \quad C_{12}C_{23} > 0,$$

where

$$Q_1 = \left| |1 + C_{12} + C_{13} + C_{23}| - |1 + C_{12} - C_{13} - C_{23}| \right| - \left| |1 - C_{12} - C_{13} + C_{23}| - |1 - C_{12} + C_{13} - C_{23}| \right|,$$

and Q_2 and Q_3 are obtained by permuting the four $\mathcal{M}_{\alpha\beta\gamma}$ in Q_1 . The correlations can be decomposed in a convex mixture of a tripartite Mermin-box and a Bell-local box which has $Q = 0$,

$$P = C_{\min}^a \left[\frac{P_{Sv}^{0001} + P_{Sv}^{1111}}{2} \right] + (1 - C_{\min}^a) P_L^{Q=0}. \quad (48)$$

The above correlations exhibit three-way contextuality if $C_{12} + C_{13} + C_{23} > 1$, however, they have nonzero tripartite Mermin discord if the state is genuinely entangled. Thus, nonzero Mermin discord of the local correlations in Eq. (48) originates from three-way contextuality.

C. Mixture of GHZ state with white noise

Here we study the correlations arising from the following Werner states [29],

$$\rho_W = p|\psi_{GHZ}\rangle\langle\psi_{GHZ}| + (1-p)\frac{\mathbb{1}}{8}, \quad (49)$$

where $|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$. The above Werner states are separable iff $p \leq 0.2$, biseparable iff $0.2 < p \leq 0.429$ and genuinely entangled iff $p > 0.429$ [30]. Notice that the Werner state have the component of the irreducible GHZ state, p , even

if the state is separable. We show that the Werner states can give rise to Svetlichny/Mermin discord if $p > 0$. Thus, the separable and biseparable states that have an irreducible genuinely entangled state component are genuinely nonclassical states as they can give rise to Svetlichny/Mermin discord.

1. Svetlichny discordant box

For the settings in Eq. (37), the Werner states give rise to the isotropic Svetlichny-box,

$$P = \frac{p}{\sqrt{2}} P_{Sv}^{0000} + \left(1 - \frac{p}{\sqrt{2}}\right) P_N. \quad (50)$$

These correlations admit the local deterministic model if $p \leq \frac{1}{\sqrt{2}}$ and have Svetlichny discord $\mathcal{G} = 4p\sqrt{2}$. Due to the component of the irreducible GHZ state and the incompatible measurements, the local correlations arising from the Werner states have genuine nonclassicality originating from Svetlichny nonlocality if $p > 0$.

2. Mermin discordant box

For the settings in Eq. (39) which gives maximal Mermin discord for the GHZ-class states, the Werner states give rise to the isotropic Mermin-box,

$$P = p \left(\frac{P_{Sv}^{0000} + P_{Sv}^{1110}}{2} \right) + (1-p)P_N. \quad (51)$$

These correlations have Mermin discord $Q = 4p > 0$ if the state has the irreducible GHZ state component. The correlations do not violate a Mermin inequality if $p \leq \frac{1}{2}$, however, they have genuine nonclassicality originating from three-way contextuality if $p > 0$.

D. Biseparable W class state

Consider the following biseparable state,

$$\rho = \frac{1}{3} |\psi_{bi}^{AB}\rangle\langle\psi_{bi}^{AB}| + \frac{1}{3} |\psi_{bi}^{AC}\rangle\langle\psi_{bi}^{AC}| + \frac{1}{3} |\psi_{bi}^{BC}\rangle\langle\psi_{bi}^{BC}|, \quad (52)$$

$|\psi_{bi}^{AB}\rangle = \frac{1}{\sqrt{2}}(|100\rangle + |010\rangle)$, $|\psi_{bi}^{AC}\rangle = \frac{1}{\sqrt{2}}(|100\rangle + |001\rangle)$ and $|\psi_{bi}^{BC}\rangle = \frac{1}{\sqrt{2}}(|010\rangle + |001\rangle)$. Svetlichny/Mermin discord for the above biseparable state can be achieved only for the suitable settings that lie in the xz -plane, for instance, the settings given in Eq. (38) gives rise to Svetlichny discord $\mathcal{G} = \frac{4\sqrt{2}}{3}$. The correlation can be decomposed as follows,

$$P = \frac{1}{3} \left[\frac{1}{\sqrt{2}} P_{PR}^{011} + \left(1 - \frac{1}{\sqrt{2}}\right) P_N^{AB} \right] P_C + \frac{1}{3} \left[\frac{P_{PR}^{001} + P_{PR}^{111}}{2} \right] P_B + \frac{1}{3} P_A \left[\frac{1}{\sqrt{2}} P_{PR}^{101} + \left(1 - \frac{1}{\sqrt{2}}\right) P_N \right], \quad (53)$$

where $P_A = P(a_m|A_i)$, $P_B = P(b_n|B_j)$ and $P_C = P(c_o|C_k)$ are the distributions arising from the state $|0\rangle$. Notice that the correlation arising from the state in Eq. (52) does not have Svetlichny/Mermin discord for all the settings that lie in the xy -plane as the state belongs to biseparable W class, i.e., the state can be written as a convex mixture of an irreducible genuinely entangled state that belongs to the W-class and a state which cannot give rise to Svetlichny/Mermin discord.

E. Mixture of GHZ state and W state

Consider the correlations arising from the following states,

$$\rho = p |\psi_{GHZ}\rangle \langle \psi_{GHZ}| + q |\psi_W\rangle \langle \psi_W|. \quad (54)$$

where $|\psi_W\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$. Since the optimal settings that gives maximal Svetlichny/Mermin discord for the GHZ state does not give nonzero Svetlichny/Mermin discord for the W-state and vice versa, Svetlichny/Mermin discord for these states arise from the component of the GHZ state or the W state for the four settings given in Eqs. (37)-(40).

For the settings in Eq. (37), the correlations have Svetlichny discord $\mathcal{G} = 4\sqrt{2}p$ and admit the following decomposition,

$$P = p \left[\frac{1}{\sqrt{2}} P_{SV}^{0000} + \left(1 - \frac{1}{\sqrt{2}}\right) P_N \right] + q P_{SVL}^{\mathcal{G}=0}, \quad (55)$$

where $P_{SVL}^{\mathcal{G}=0}$ is a Svetlichny-local box arising from the W state which has zero Svetlichny discord.

For the settings in Eq. (38), the correlations have Svetlichny discord $\mathcal{G} = \frac{8\sqrt{2}q}{3}$ and admit the following decomposition,

$$P = p P_{NL}^{\mathcal{G}=0}(\psi_{GHZ}) + q P_{NL}^{\mathcal{G}>0}(\psi_W), \quad (56)$$

where $P_{NL}^{\mathcal{G}=0}(\psi_{GHZ})$ is the three-way nonlocal box arising from the GHZ state given in Eq. (45) and $P_{NL}^{\mathcal{G}>0}(\psi_W)$ is the three-way nonlocal box arising from the W state given in Eq. (47) with $C_{min}^a = \frac{2}{3}$.

As the correlations in Eq. (56) violates the class 99 facet inequality, they do not belong to the Svetlichny-box polytope. However, the correlations in Eq. (55) belong to the three-way nonlocal region of the Svetlichny-box polytope.

F. Classical-quantum, quantum-classical and genuinely quantum-correlated states

A mixed three-qubit state can give rise to Svetlichny discord or Mermin discord iff all the three qubits are nonclassically correlated. The states that do not have Svetlichny discord and Mermin discord can be decomposed in the form of classical-quantum or quantum-classical states defined as follows.

Definition 7. The classical-quantum (CQ) states can be decomposed as,

$$\rho_{CQ}^{1|23} = \sum_i p_i \rho_i^A \otimes \rho_i^{BC}, \quad (57)$$

whereas the quantum-classical (QC) states can be decomposed as,

$$\rho_{QC}^{12|3} = \sum_i p_i \rho_i^{AB} \otimes \rho_i^C \quad (58)$$

or

$$\rho_{QC}^{13|2} = \sum_i p_i \rho_i^{AC} \otimes \rho_i^B, \quad (59)$$

where ρ_i^{AB} , ρ_i^{AC} , and ρ_i^{BC} are, in general, quantum-correlated states which are neither classical-quantum nor quantum-classical states [10] and there is no restriction on ρ_i^A , ρ_i^B , and ρ_i^C .

Theorem 2. All CQ and QC states given in Eqs. (57)-(59) have $\mathcal{G} = \mathcal{Q} = 0$ for all measurements.

Proof. Consider the QC states as given in Eq. (58). For these states, the expectation value factorizes as follows,

$$\langle A_i B_j C_k \rangle = \sum_i p_i \langle A_i B_j \rangle_i \langle C_k \rangle_i, \quad (60)$$

which implies that the Svetlichny operators in \mathcal{G}_1 factorize as follows,

$$\begin{aligned} \mathcal{G}_1 = & \left| \sum_i p_i \{ \mathcal{B}_{000}^i \langle C_0 \rangle_i + \mathcal{B}_{111}^i \langle C_1 \rangle_i \} - \sum_i p_i \{ \mathcal{B}_{000}^i \langle C_0 \rangle_i - \mathcal{B}_{111}^i \langle C_1 \rangle_i \} \right| \\ & - \left| \sum_i p_i \{ \mathcal{B}_{000}^i \langle C_1 \rangle_i + \mathcal{B}_{111}^i \langle C_0 \rangle_i \} - \sum_i p_i \{ \mathcal{B}_{000}^i \langle C_1 \rangle_i - \mathcal{B}_{111}^i \langle C_0 \rangle_i \} \right| \\ & - \left| \sum_i p_i \{ \mathcal{B}_{010}^i \langle C_0 \rangle_i + \mathcal{B}_{100}^i \langle C_1 \rangle_i \} - \sum_i p_i \{ \mathcal{B}_{010}^i \langle C_0 \rangle_i - \mathcal{B}_{100}^i \langle C_1 \rangle_i \} \right| \\ & - \left| \sum_i p_i \{ \mathcal{B}_{010}^i \langle C_0 \rangle_i + \mathcal{B}_{100}^i \langle C_1 \rangle_i \} - \sum_i p_i \{ \mathcal{B}_{010}^i \langle C_1 \rangle_i - \mathcal{B}_{100}^i \langle C_0 \rangle_i \} \right|. \end{aligned} \quad (61)$$

Here $\mathcal{B}_{\alpha\beta\gamma}^i$ which are the Bell functions in the following CHSH inequalities [31],

$$\mathcal{B}_{\alpha\beta\gamma} := (-1)^\gamma \langle A_0 B_0 \rangle + (-1)^{\beta\oplus\gamma} \langle A_0 B_1 \rangle + (-1)^{\alpha\oplus\gamma} \langle A_1 B_0 \rangle + (-1)^{\alpha\oplus\beta\oplus\gamma\oplus 1} \langle A_1 B_1 \rangle \leq 2, \quad (62)$$

and $\langle C_k \rangle_i$ are evaluated for ρ_{AB}^i and ρ_C^i given in Eq. (58). Let us now try to maximize \mathcal{G}_1 with respect to the quantum-classical states in which ρ_{AB}^i are the quantum-correlated states. For an optimal settings that gives nonzero for only one of $\mathcal{B}_{\alpha\beta\gamma}^i$ in Eq. (61), $\mathcal{G}_1 = 0$. Similarly, we can prove that $Q = 0$ by exploiting the factorization property in Eq. (60).

Since \mathcal{G} and Q are symmetric under the permutations of the parties, they are also zero for the states in Eqs. (57) and (59) for all measurements. \square

All the genuinely entangled states are only a subset of the set of nonclassical states with respect to \mathcal{G} and Q . The nonclassical biseparable and separable states are the genuinely quantum-correlated states.

Definition 8. A genuinely quantum-correlated state cannot be written in the classical-quantum or quantum-classical form given in Eqs. (57) -(59) and admits the following decomposition,

$$\rho = p_1 \sum_i q_i \rho_i^A \otimes \rho_i^{BC} + p_2 \sum_j r_j \rho_j^{AC} \otimes \rho_j^B + p_3 \sum_k s_k \rho_k^{AB} \otimes \rho_k^C, \quad (63)$$

with atleast two of the three coefficients p_1 , p_2 , and p_3 are nonzero.

V. CONCLUSIONS

We have introduced the measures, Svetlichny and Mermin discord, to characterize the presence of genuine nonclassicality in probability distributions arising from tripartite quantum states. By using these measures, we have obtained a 3-decomposition of any box in the Svetlichny-box polytope which is a straightforward generalization of the bipartite PR-box polytope to the multipartite case. This decomposition allows us to isolate the origin of genuine nonclassicality into three disjoint sources: a Svetlichny box which exhibits three-way nonlocality, a maximally two-way nonlocal box which exhibits three-way contextuality and a classical correlation. We find that the Svetlichny-box polytope does not characterize all genuinely three-way nonlocal quantum correlations.

We have applied Svetlichny and Mermin discord to the correlations arising from various genuinely nonclassical three-qubit states for projective measurements. For these states, we find that a nonzero Svetlichny and Mermin discord originate from noncommuting measurements that give rise to

Svetlichny nonlocality and three-way contextuality, respectively. In the case of pure states, Svetlichny and Mermin discord can be nonzero iff the given state is genuinely entangled. Moving to the mixed states, Svetlichny/Mermin discord detects the component of an irreducible genuinely entangled state in the decomposition of the biseparable and fully separable states. If a mixed state admits a decomposition that has an irreducible GHZ-class state and W-class state components simultaneously, nonzero Svetlichny/Mermin discord originates from the GHZ-class state or the W-class state. We find that when the GGHZ states and Werner states give rise optimal Svetlichny or Mermin discord, irreducible GHZ state component of the Werner states plays a role analogous to entanglement of the GGHZ states.

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Appendix A: Mermin box

The observation of Mermin boxes with maximally mixed marginals does not necessarily imply the presence of a three-qubit GHZ state. They can also arise from a six-qubit 4-separable state. For instance, consider the following entangled state,

$$\rho = \frac{1}{4} (|0\rangle\langle 0|_x \otimes |\phi^+\rangle\langle \phi^+| + |1\rangle\langle 1|_x \otimes |\phi^-\rangle\langle \phi^-|) \otimes (|0\rangle\langle 0|_y \otimes |\psi^+\rangle\langle \psi^+| + |1\rangle\langle 1|_y \otimes |\psi^-\rangle\langle \psi^-|), \quad (A1)$$

where $|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm i|10\rangle)$ with Alice holding the first and fourth qubit, Bob the second and fifth and Charlie the third and sixth. Suppose Alice chooses $A_0 = \sigma_x$, she measures it on the first qubit, and, Bob and Charlie measure $B_0 = \sigma_x$, $B_1 = \sigma_y$, $C_0 = \sigma_x$ and $C_1 = \sigma_y$ on the second and third qubit. Suppose Alice chooses $A_1 = \sigma_y$, she measures it on the fourth qubit, and, Bob and Charlie measure $B_0 = \sigma_x$, $B_1 = \sigma_y$, $C_0 = \sigma_x$ and $C_1 = \sigma_y$ on the fifth and sixth qubit. It can be checked that for this measurement strategy, the 4-separable state in Eq. (A1) gives rise to the Mermin box in Eq. (22) and the following entangled state

$$\rho = \frac{1}{4} (|0\rangle\langle 0|_x \otimes |\phi^+\rangle\langle \phi^+| + |1\rangle\langle 1|_x \otimes |\phi^-\rangle\langle \phi^-|) \otimes |0\rangle\langle 0|_y \otimes |\psi^+\rangle\langle \psi^+|, \quad (A2)$$

gives rise to the Mermin box with a nonmaximally mixed marginal in Eq. (25).

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